

IC/94/372  
hep-ph/9502282

International Atomic Energy Agency  
and  
United Nations Educational Scientific and Cultural Organization  
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

**CURRENT  $s$  - QUARK MASS CORRECTIONS TO THE FORM  
FACTORS OF  $D$  - MESON SEMILEPTONIC DECAYS**

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**Abstract**

The infinite mass effective theory, when a heavy quark mass tends to infinity, and Chiral perturbation theory at the quark level, based on the extended Nambu – Jona – Lasinio model with linear realization of chiral  $U(3) \times U(3)$  symmetry, are applied to the calculation of current  $s$  – quark mass corrections to the form factors of the  $D \rightarrow \bar{K} e^+ \nu_e$  and  $D \rightarrow \bar{K}^* e^+ \nu_e$  decays. These corrections turn out to be quite significant, of the order of 7 – 20%. The theoretical results are compared with experimental data.

MIRAMARE – TRIESTE  
November 1994

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# 1 Introduction

In our recent publications [1,2] we calculated in the chiral limit the form factors of the semileptonic  $D \rightarrow \bar{K}^* e^+ \nu_e$  and  $D \rightarrow \bar{K} e^+ \nu_e$  decays. For the description of  $D$ -mesons we applied the infinite mass effective theory (IMET) [3,4], when the  $c$ -quark mass  $M_c$  tends to infinity, i.e.  $M_c \rightarrow \infty$ . In IMET the long – distance physics we describe within Chiral perturbation theory at the quark level (CHPT) <sub>$q$</sub>  [5], based on the extended Nambu – Jona – Lasinio (ENJL) model with linear realization of chiral  $U(3) \times U(3)$  symmetry [6].

In this paper we apply IMET and (CHPT) <sub>$q$</sub>  to the calculation of the fine structure of the form factors of the  $D \rightarrow \bar{K} e^+ \nu_e$  and  $D \rightarrow \bar{K}^* e^+ \nu_e$  decays at the first order in current  $s$  – quark mass expansion. Within IMET and (CHPT) <sub>$q$</sub>  the first order current – quark – mass corrections to the mass spectra of charmed pseudoscalar and vector mesons and charmed pseudoscalar – meson leptonic constants have been calculated in [7]. The amplitude of the  $D \rightarrow h e^+ \nu_e$  decay can be determined as follows

$$M(D \rightarrow h e^+ \nu_e) = -\frac{G_F}{\sqrt{2}} V_{cs}^* < h(Q) | \bar{s}(0) \gamma_\mu (1 - \gamma^5) c(0) | D(p) > \ell^\mu, \quad (1)$$

where  $h = \bar{K}$  or  $\bar{K}^*$ ,  $G_F = 1.166 \times 10^{-5}$  GeV $^{-2}$  is Fermi weak constant,  $|V_{cs}| = 0.975$  is the CKM – mixing matrix element,  $s(0)$  and  $c(0)$  are the  $s$  – and  $c$  – current quark fields with  $N$  colour degrees of freedom each, and  $\ell^\mu = \bar{u}(k_{\nu_e}) \gamma^\mu (1 - \gamma^5) v(k_{e^+})$  is the weak leptonic current.

We shall seek the hadronic matrix element

$$M_\mu(D \rightarrow h) = < h(Q) | \bar{s}(0) \gamma_\mu (1 - \gamma^5) c(0) | D(p) > \quad (2)$$

in the form of an expansion in powers of the current  $s$  – quark mass upto first order terms

$$M_\mu(D \rightarrow h) = M_\mu^{(0)}(D \rightarrow h) + M_\mu^{(1)}, (D \rightarrow h). \quad (3)$$

Here we have denoted

$$M_\mu^{(0)}(D \rightarrow h) = < h(Q) | \bar{s}(0) \gamma_\mu (1 - \gamma^5) c(0) | D(p) >_{\text{ch.l.}} \quad (4)$$

$$\begin{aligned} M_\mu^{(1)}(D \rightarrow h) &= \\ &= -i m_0 s \int d^4 x < h(Q) | T(\bar{s}(x) s(x) \bar{s}(0) \gamma_\mu (1 - \gamma^5) c(0)) | D(p) >_{\text{ch.l.}} . \end{aligned} \quad (5)$$

The matrix element  $M_\mu^{(0)}(D \rightarrow h)$  describes the  $D \rightarrow h$  transition calculated in the chiral limit (ch.l.) while  $M_\mu^{(1)}(D \rightarrow h)$  is the first order correction in the current  $s$  – quark mass expansion. The matrix elements  $M_\mu^{(0)}(D \rightarrow h)$  for  $h = \bar{K}^*$  and  $\bar{K}$  have been calculated in [1,2]. In this paper we shall calculate  $M_\mu^{(1)}(D \rightarrow h)$ .

In accordance to the procedure expounded in [1,2,7] we reduce (5) to the expression

$$\begin{aligned} M_\mu^{(1)}(D \rightarrow h) &= g_D m_0 s i \int d^4 x, \int_{-\infty}^{\infty} dz_0 \theta(-z_0) \times \\ &\times < h(Q) | T(\bar{s}(x) s(x) s(0) \gamma_\mu \left( \frac{1 + \gamma^0}{2} \right) \gamma^5 q(z_0, \vec{0})) | 0 >_{\text{ch.l.}} \end{aligned} \quad (6)$$

obtained in leading order in the large  $N$  and  $M_c$  expansion,  $q = u$  or  $d$  for  $D^0$  or  $D^+$ , respectively. The coupling constant  $g_D$  has been calculated in [8]

$$g_D = \frac{2\sqrt{2}\pi}{\sqrt{N}} \left( \frac{M_D^2}{M_c \bar{v}'} \right)^{1/2}, \quad (7)$$

where  $\bar{v}' = 4\Lambda = 2.66$  GeV and  $\Lambda$  is the cut – off in 3 – dimensional Euclidean momentum space.  $\Lambda$  is connected to the scale of spontaneous breaking of chiral symmetry (SBCS)  $\Lambda_\chi$  by the relation  $\Lambda = \Lambda_\chi/\sqrt{2} = 0.66$  GeV at  $\Lambda_\chi = 0.94$  GeV [5].

The r.h.s. of (6) involves only the light – quark fields. Therefore for the evaluation of (6) we can apply  $(\text{CHPT})_q$  [5,7]. Since the leading order of the r.h.s. of (6) in current – quark – mass expansion is fixed by the factor  $m_{0s}$ , so the matrix element  $\langle h(Q)|T(\dots)|0\rangle$  has to be calculated in the chiral limit (ch.l.).

By applying the formulas of quark conversion (Ivanov [5]) we can present the matrix element  $M_\mu^{(1)}(D \rightarrow h)$  in terms of constituent – quark – loop diagrams [7]. The momentum representation of these diagrams reads

$$\begin{aligned} M_\mu^{(1)}(D \rightarrow h) &= i m_{0s} g_D g_h \frac{\bar{v}}{4m} \left( -\frac{N}{16\pi^2} \right) \int \frac{d^4k}{\pi^2 i} \text{tr} \left[ \frac{1}{m - \hat{k}} \Gamma_h \times \right. \\ &\times \left. \frac{1}{m - \hat{Q} - \hat{k}} \frac{1}{m - \hat{Q} - \hat{k}} \gamma_\mu (1 - \gamma^5) \left( \frac{1 + \hat{v}}{2} \right) \gamma^5 \frac{1}{k \cdot v + i0} \right]. \end{aligned} \quad (8)$$

The appearance of the factor  $\bar{v}/4m$  is due to the contribution of the diagram with the light scalar  $\sigma_s$  – meson exchange [8]. Here  $\bar{v} = -\langle 0|\bar{q}q|0\rangle/F_0^2 = 1.92$  GeV,  $F_0 = 0.092$  GeV and  $m = 0.33$  GeV are the PCAC constant of light pseudoscalar mesons and the constituent quark mass calculated in the chiral limit [5]. The coupling constants  $g_h$  describe the interaction between light constituent quarks and light mesons  $\bar{K}$  and  $\bar{K}^*$ , that is  $g_{\bar{K}} = 2\pi/\sqrt{N}$  and  $g_{\bar{K}^*} = \pi\sqrt{6}/\sqrt{N}$  [7,8] such that  $g_{\bar{K}^*}/g_{\bar{K}} = \sqrt{3/2}$  [5].  $\Gamma_h$  is either  $\Gamma_{\bar{K}} = i\gamma^5$  or  $\Gamma_{\bar{K}^*} = \gamma_\nu e^{*\nu}(Q)$  depending on whether  $h = \bar{K}$  or  $h = \bar{K}^*$ . Now we can proceed to the calculation of the current  $s$  – quark mass corrections to the form factors.

## 2 The $D \rightarrow \bar{K} e^+ \nu_e$ decay

For the  $D \rightarrow \bar{K}$  the matrix element  $M_\mu^{(1)}(D \rightarrow \bar{K})$  reads

$$\begin{aligned} M_\mu^{(1)}(D \rightarrow \bar{K}) &= i m_{0s} g_D \frac{2\pi}{\sqrt{N}} \frac{\bar{v}}{4m} \left( -\frac{N}{16\pi^2} \right) \int \frac{d^4k}{\pi^2 i} \text{tr} \left[ \frac{1}{m - \hat{k}} i\gamma^5 \times \right. \\ &\times \left. \frac{1}{m - \hat{Q} - \hat{k}} \frac{1}{m - \hat{Q} - \hat{k}} \gamma_\mu (1 - \gamma^5) \left( \frac{1 + \hat{v}}{2} \right) \gamma^5 \frac{1}{k \cdot v + i0} \right] = \\ &= m_{0s} g_D \frac{\sqrt{N}}{4\pi} \int \frac{d^4k}{\pi^2 i} \frac{k_\mu}{[m^2 - k^2 - i0][m^2 - (k + Q)^2 - i0]} \frac{1}{k \cdot v + i0} + \\ &+ \dots \end{aligned} \quad (9)$$

Following [9,10] we have kept only divergent contributions. The dots denote the contributions of convergent integrals. The integration over  $k$  gives [1]

$$\begin{aligned} & \int \frac{d^4 k}{\pi^2 i} \frac{k_\mu}{[m^2 - k^2 - i0][m^2 - (k + Q)^2 - i0]} \frac{1}{k \cdot v + i0} = \\ & = v_\mu 2 \ln \left( 1 + \frac{\bar{v}'}{4Q_0} \right) + Q_\mu \frac{2}{Q_0} \left[ 1 - \ln \left( 1 + \frac{\bar{v}'}{4Q_0} \right) \right] \end{aligned} \quad (10)$$

where  $Q_0 = (M_D^2 - q^2)/2M_D$  is the energy of the massless  $K$  – meson in the rest frame of the  $D$  – meson. The appearance of the  $q^2$  – dependence is due to the  $q^2$  – dependence of  $Q_0$ . The matrix element  $M_\mu^{(1)}(D \rightarrow \bar{K})$  can be expressed in terms of two form factors

$$M_\mu^{(1)}(D \rightarrow \bar{K}) = f_+^{(1)}(q^2)(p + Q)_\mu + f_-^{(1)}(q^2),(p - Q)_\mu \quad (11)$$

where

$$\begin{aligned} f_+^{(1)}(q^2) &= \frac{m_{0s}}{M_*} \frac{\bar{v}}{4m} \frac{M_D^2}{M_D^2 - q^2} \left[ 1 - \frac{M_D^2 + q^2}{2M_D^2} \ln \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right], \\ f_-^{(1)}(q^2) &= -\frac{m_{0s}}{M_*} \frac{\bar{v}}{4m} \frac{M_D^2}{M_D^2 - q^2} \left[ 1 - \frac{3M_D^2 - q^2}{2M_D^2} \ln \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right]. \end{aligned} \quad (12)$$

Here we have denoted  $2M_* = \sqrt{2M_D \bar{v}'}$ . It should be stressed that the formulae (12) are valid in the physical region only, i.e.  $0 \leq q^2 \leq (M_D - m_K)^2$ . At  $q^2 = 0$  the form factors  $f_+^{(1)}(0)$  and  $f_-^{(1)}(0)$  read

$$\begin{aligned} f_+^{(1)}(0) &= \frac{m_{0s}}{M_*} \frac{\bar{v}}{4m} \left[ 1 - \frac{1}{2} \ln \left( 1 + \frac{M_*^2}{M_D^2} \right) \right] = 0.09, \\ f_-^{(1)}(0) &= -\frac{m_{0s}}{M_*} \frac{\bar{v}}{4m} \left[ 1 - \frac{3}{2} \ln \left( 1 + \frac{M_*^2}{M_D^2} \right) \right] = -0.02. \end{aligned} \quad (13)$$

In the chiral limit the quantity  $f_+(0)$  has been calculated in [8] (see also [2])

$$f_+^{(0)}(0) = \frac{1}{\sqrt{2}} \left( \frac{\bar{v}'}{2M_c} \right)^{1/2} = 0.6. \quad (14)$$

The numerical value is estimated at the equality  $M_c = M_D = 1.86$  GeV accepted in our approach [9]. By adding the current  $s$  – quark mass correction (13) we get the total value of  $f_+(0)$

$$f_+(0) = f_+^{(0)}(0) + f_+^{(1)}(0) = 0.69 \quad (15)$$

which is good compared with the experimental data  $|f_+(0)|_{\text{exp}} = 0.7 \pm 0.1$  [11]. Our result  $f_+(0) = 0.69$  agrees well with the theoretical prediction by Dominguez and Paver [12] obtained within the QCD sum rule approach. We find the current s-quark mass correction to be about 15%.

### 3 The $D \rightarrow \bar{K}^* e^+ \nu_e$ decay

The matrix element  $M_\mu^{(1)}(D \rightarrow \bar{K}^*)$  can be expressed in terms of four form factors [1]

$$\begin{aligned} M_\mu^{(1)}(D \rightarrow \bar{K}^*) = & i a_1^{(1)}(q^2) e_\mu^*(Q^2) - i a_2^{(1)}(q^2) (e^*(Q) \cdot p) (p + Q)_\mu - \\ & - i a_3^{(1)}(q^2) (e^*(Q) \cdot p) (p - Q)_\mu - \\ & - 2 b^{(1)}(q^2) \varepsilon_{\mu\nu\alpha\beta} e^{t\nu}(Q) p^\alpha Q^\beta, (\varepsilon^{0123} = 1). \end{aligned} \quad (16)$$

In order to obtain the form factors  $a_i^{(1)}(q^2)$  ( $i = 1, 2, 3$ ) and  $b^{(1)}(q^2)$  we have to calculate the following momentum integral

$$\begin{aligned} \mathcal{M}_{\mu\nu} = & \int \frac{d^4 k}{\pi^2 i} \text{tr} \left[ \frac{1}{m - \hat{k}} \gamma_\nu \frac{1}{m - \hat{Q} - \hat{k}} \frac{1}{m - \hat{Q} - \hat{k}} \times \right. \\ & \times \gamma_\mu (1 - \gamma^5) \left( \frac{1 + \hat{v}}{2} \right) \gamma^5 \frac{1}{k \cdot v + i 0} \left. \right]. \end{aligned} \quad (17)$$

By keeping only divergent contributions [9,10] and using the integrals given in the Appendix of [1], we get

$$\begin{aligned} \mathcal{M}_{\mu\nu} = & - 4 \ell n \left( \frac{\bar{v}'}{4m} \right) g_{\mu\nu} + \frac{8}{M_D^2 - q^2} \left[ 1 - \ell n \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right] Q_\mu p_\nu - \\ & - \frac{8i}{M_D^2 - q^2} \left[ 1 - \ell n \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right] \varepsilon_{\mu\nu\alpha\beta} p^\alpha Q^\beta. \end{aligned} \quad (18)$$

The appearance of the  $q^2$  – dependence is due to the quantity  $Q_0 = (M_D^2 - q^2)/2M_D$ , being the energy of the massless  $\bar{K}^*$  – meson in the rest frame of the  $D$  – meson. The neglect of the  $\bar{K}^*$  – meson mass in the r.h.s. of (17) is in accordance with the prescription of  $(\text{CHPT})_q$  which incorporates the Vector Dominance approach [5,13], admitting the smooth dependence of low – energy hadronic matrix elements on the masses of low – lying vector mesons ( $\rho, \omega, \varphi, K^*$ ) [1,13,14].

By using (18), one can calculate the following chiral corrections to the form factors of the  $D \rightarrow \bar{K}^*$  transition

$$\begin{aligned} a_1^{(1)}(q^2) &= \frac{\sqrt{3}}{2} \frac{m_{0s}}{M_*} \frac{\bar{v}}{4m} M_D \ell n \left( \frac{\bar{v}'}{4m} \right) \\ a_2^{(1)}(q^2) &= -a_3^{(1)}(q^2) = b^{(1)}(q^2) \\ b^{(1)}(q^2) &= \frac{\sqrt{3}}{2} \frac{m_{0s}}{M_*} \frac{\bar{v}}{4m} \frac{M_D}{M_D^2 - q^2} \left[ 1 - \ell n \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right]. \end{aligned} \quad (19)$$

In the chiral limit the form factors of the  $D \rightarrow \bar{K}^*$  transition have been calculated in [1]

$$\begin{aligned}
a_1^{(0)}(q^2) &= \sqrt{\frac{3}{8}} M_* \\
a_2^{(0)}(q^2) &= \sqrt{\frac{3}{8}} \frac{M_*}{M_D^2} \left[ \frac{q^2}{M_D^2 - q^2} + \right. \\
&\quad \left. + \frac{M_D^2 - q^2}{M_*^2} \left( 1 - \frac{2mM_D}{M_D^2 - q^2} \right) \ln \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right], \\
a_3^{(0)}(q^2) &= -\sqrt{\frac{3}{8}} \frac{M_*}{M_D^2} \left[ \frac{2M_D^2 - q^2}{M_D^2 - q^2} - \right. \\
&\quad \left. - \frac{M_D^2 - q^2}{M_*^2} \left( 1 - \frac{2mM_D}{M_D^2 - q^2} \right) \ln \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right], \\
b^{(0)}(q^2) &= \sqrt{\frac{3}{8}} \frac{1}{M_*} \ln \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right).
\end{aligned} \tag{20}$$

Here we have used the relation (14). The numerical values of the form factors at  $q^2 = 0$  read

$$\begin{aligned}
a_1(0) &= a_1^{(0)}(0) + a_1^{(1)}(0) = 0.96 + 0.14 = 1.10 \text{ (GeV)}, \\
a_2(0) &= a_2^{(0)}(0) + a_2^{(1)}(0) = 0.14 + 0.03 = 0.17 \text{ (GeV)}^{-1}, \\
a_3(0) &= a_3^{(0)}(0) + a_3^{(1)}(0) = -0.42 - 0.03 = -0.45 \text{ (GeV)}^{-1}, \\
b(0) &= b^{(0)}(0) + b^{(1)}(0) = 0.21 + 0.03 = 0.24 \text{ (GeV)}^{-1}.
\end{aligned} \tag{21}$$

One sees that the first order current s-quark mass mass corrections are between 7 and 20%. The form factors  $a_i(q^2)$  ( $i = 1, 2, 3$ ) and  $b(q^2)$  are connected with the standard form factors  $A_i(q^2)$  ( $i = 1, 2, 3$ ) and  $V(q^2)$  via the relations [1]

$$\begin{aligned}
A_1(q^2) &= \frac{1}{M_D + M_{\bar{K}^*}} a_1(q^2)|_{q^2=0} = 0.40 \\
A_2(q^2) &= (M_D + M_{\bar{K}^*}) a_2(q^2)|_{q^2=0} = 0.47 \\
A_3(q^2) &= (M_D + M_{\bar{K}^*}) a_3(q^2)|_{q^2=0} = -1.24 \\
V(q^2) &= (M_D + M_{\bar{K}^*}) b(q^2)|_{q^2=0} = 0.66,
\end{aligned} \tag{22}$$

where  $M_{\bar{K}^*} = 0.89$  GeV is the mass of the  $\bar{K}^*$  – meson [11]. The theoretical values compare reasonably with recent experimental data [15]

$$\begin{aligned}
A_1(0)_{exp} &= 0.46 \pm 0.05 \pm 0.05, \\
A_2(0)_{exp} &= 0.38 \pm^{0.11}_{0.12} \pm 0.07, \\
V(0)_{exp} &= 0.92 \pm^{0.19}_{0.18} \pm 0.12.
\end{aligned} \tag{23}$$

These numerical results obtained by taking into account the first order current  $s$  – quark mass corrections confirm the results found in [1]. It is because in [1] we expressed the form factors of the  $D \rightarrow \bar{K}^*$  transition in terms of the form factor of the  $D \rightarrow \bar{K}$  transition  $f_+(0)$ . There for the numerical estimate we used the value  $f_+(0) = 0.7$ , which we obtained in present paper only at the first order in current  $s$  – quark mass

expansion (15). Recall that in the chiral limit we have  $f_+(0) = 0.6$ . This overlap of results underscores the self – consistency of the current  $s$  – quark mass corrections to the form factors of the transitions  $D \rightarrow \bar{K}^*$  and  $D \rightarrow \bar{K}$  calculated within IMET and  $(\text{CHPT})_q$ .

## 4 Conclusion

We have applied IMET and  $(\text{CHPT})_q$  for the computation of the current  $s$  – quark mass corrections to the form factors of the semileptonic decays of the non – strange charmed  $D$  – mesons,  $D \rightarrow \bar{K} e^+ \nu_e$  and  $D \rightarrow \bar{K}^* e^+ \nu_e$ . We have obtained non – zero contributions for the first order corrections in current  $s$  – quark mass expansion to the form factors of the  $D \rightarrow \bar{K} e^+ \nu_e$  decays. This result contradicts the Ademollo – Gato theorem for the form factors of the semileptonic decays of  $K$  – mesons [16]. Within  $(\text{CHPT})_q$  the Ademollo – Gato theorem has been analyzed in [17]. The observed contradiction can be explained as an effect of IMET. Indeed IMET is based on the infinite limit  $M_c \rightarrow \infty$  which violates chiral  $SU(4) \times SU(4)$  symmetry, a necessary condition for the validity of the Ademollo – Gato theorem for the the  $D \rightarrow \bar{K} e^+ \nu_e$  decays. The current  $s$  – quark mass corrections to the form factors of the  $D \rightarrow \bar{K}^* e^+ \nu_e$  decays are consistent with the corrections calculated for the form factors of the  $D \rightarrow \bar{K} e^+ \nu_e$  decays.

Note that we have kept to the first order corrections in current  $s$  – quark mass expansion calculated at the tree – meson level. Of course, the one – meson – loop corrections can be taken into account too. The consistent procedure for meson – loop chiral corrections within  $(\text{CHPT})_q$  has been developed in Ref.[5]). This procedure can also be applied to charmed meson physics.

## 5 Acknowledgements

ANI and NIT would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. With pleasure we acknowledge fruitful discussions with Prof. G. E. Rutkovsky.

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